PR2 Handbook:

How to use mathematics exhibits in the classroom

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Introduction

Mathematics is a key part of STEAM subjects and one of the main skills needed today and, in the future, to awaken scientific vocations among young people. The acronym SMEM used for this project stands for “Significant Mathematics for Early Mathematicians”. The SMEM project has taken on a multidimensional approach that aims to create a new space for innovative teaching methods in mathematics, reduce the gender gap related to STEM-oriented pathways, cultivate a variety of soft and human-centric skills and foster a positive image of mathematics as a subject. The exhibition is addressed to children aged between 3 and 8 years, as well as their teachers, but also to anyone who is interested in bridging the gap between mathematics and play.

The project is proposed from the point of view of non-formal education, which we could summarise as: "We don't teach, but they learn". As such, this creates a virtuous circle of the “Hands-on, Minds-on, Hearts-on and Talk-on” experience.

In this sense:
- the proposed activities are undirected;
- the information is based on suggestions rather than instructions;

- the main goal is not to solve the challenge, but to create a conversation and collaboration between users;
- the tasks are not explicit, leaving a large space for users to interpret the challenge presented, to choose the strategy to resolve it and to interact with the material.

In this regard, it should be mentioned that the nature of the exhibits is based on inclusive design as described by CAST’s (2018) conceptualisation of Universal Design for Learning Version 2.2, where there are multiple means of presentation, engagement and expression.

Specific tasks, especially if they present greater difficulties, can be introduced by the facilitator in the exhibition time, such as spontaneous workshops or on special occasions such as teacher training. The rationale behind the exhibits is to allow users to engage with mathematical concepts and skills through seemingly simple challenges. Moreover, through this project, we are able to verify the effectiveness of hands-on physical and virtual exhibits and evaluate the results based on engagement, competence and skill development, as well as promoting a deeper understanding of the concepts presented. These aspects are a work in progress that is open to the contribution of the educational community.

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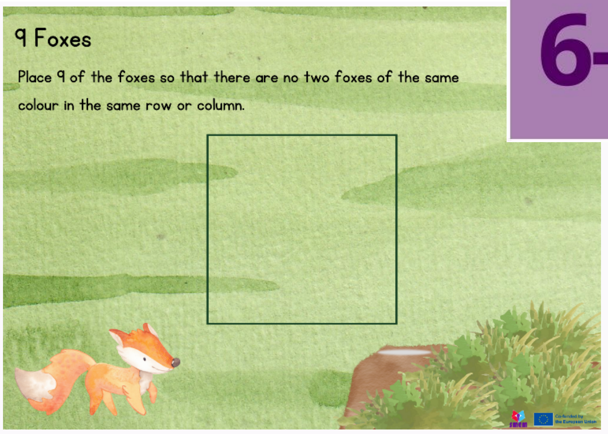
9 Foxes

Age Group
6-8



Material

3D printed board or game grid printed on paper/cardboard.
3D printed foxes in three different colours (alternatively, bought or printed tokens in 3 different colours).

Activity

The activity consists of two parts: Can you sort the foxes by colour? Can you solve the Latin square? A Latin square is a matrix of n rows and n columns) filled with n distinct elements, each row and each column containing only one copy.

Solution

First, the pupils should classify the foxes or tokens by colour. It will enable them to identify the different colours and realise they only need three distinct colours for this activity. There are three foxes of each colour, which makes nine foxes in total.
Then, the kids should solve the Latin square. This part of the activity helps students to recognize rows and columns. It also gives them a gentle introduction to the game of Sudoku and magic squares.

Further Explorations

The activity of solving the 3rd-order Latin square (which has one unique solution) might be extended to solving a 4th-order Latin square that has 576 solutions!
Optionally, you could introduce Sudoku as the next level or even play with magic squares. Start with small figures, tokens or geometric shapes, then gradually introduce numbers. If working with magic squares, you could try the task of making the sum of each row, column, and diagonal equal.

Mathematical Background

Patterns
Strategies

Transferable Skills

Colour recognition
Classifying objects according to a strict rule
Implementing a strategy to solve a complex problem
Working on spatial geometry.

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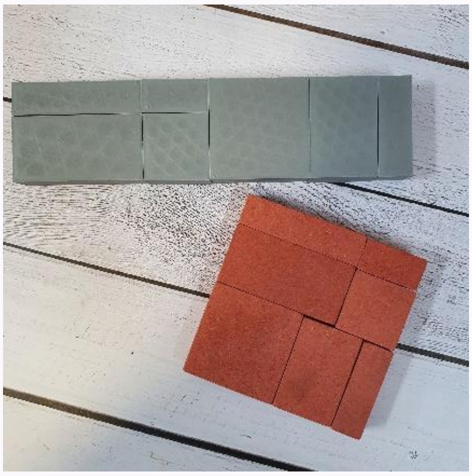
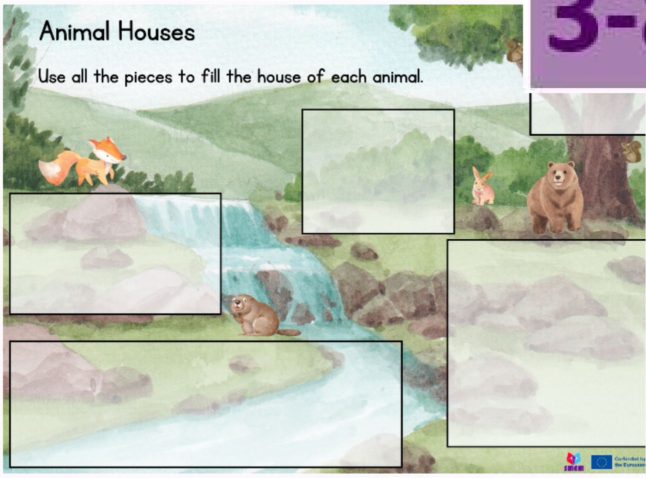
Animal Houses

Age Group
3-8





Material

The board is printed on cardboard or laminated paper.
Seven prisms made of wood or by 3D printing. Alternatively, they could be made out of thin cardboard, folded and glued. They have all one unit of thickness and different areas:

Prism no.	1	2	3	4	5	6	7
Dimensions	1x2	1x3	1x4	2x2	2x3	2x4	3x3

Activity

The instructions are not precise and allow different interpretations, not all coherent with a final solution.
For example, equal dimensions of one of the prisms (3x3x1) and the bird's house will probably inspire the kids to "build" that house with it. Without a doubt, they will notice something is wrong because all the other houses remain empty.
The suggestion to start building the bear's house (the 6x6 square) sparks the idea that all the houses must be constructed by using all the pieces, even if, in the end, it will result in houses of different thicknesses (one, two, three and even four units).
The preference is always to set a goal in the second phase.

Solution

As already pointed out, it is crucial to suggest commencing with the building process from the Bear house represented by the 6x6 square. Going this way the other houses can be solved with a minimal number of movements, thus highlighting patterns.
Next, you can build the Beaver house with a single layer of all the pieces. It is straightforward to transform the Bear house to fit the rectangle dimensions of the Beaver house.
The Bird’s house (little square with dimensions of 3x3) is the most challenging one to solve because the two prisms with sides of four units cause a counterintuitive action: assembling that house vertically.

Further Explorations

Depending on the age, it is possible to introduce small challenges:

- measure the total area of all houses (area/volume ratio);
- reflect on the equivalence of the different houses, all with a volume of 36 units;

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* find other structures with a volume of 36 units and check which ones are buildable with the given prisms and which ones require changing the dimensions of the prisms (this is easy to do by using multilink cubes).

Mathematical Background

Dimensions: areas, volumes
Combinatorics
Patterns

Transferable Skills

Observation
Classification
Trial and error analysis
Approach to computational thinking

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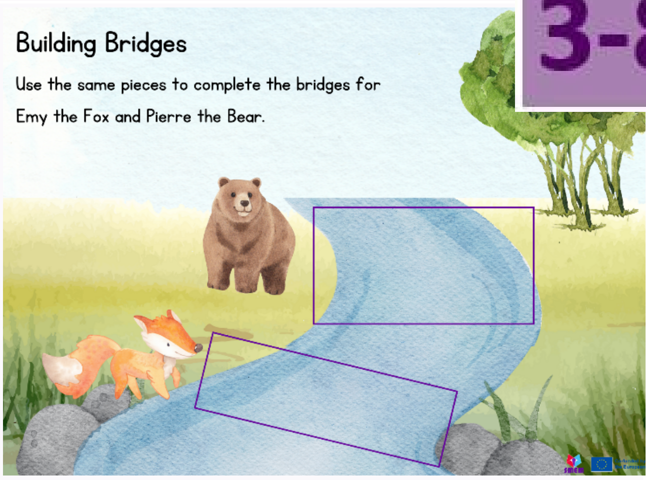
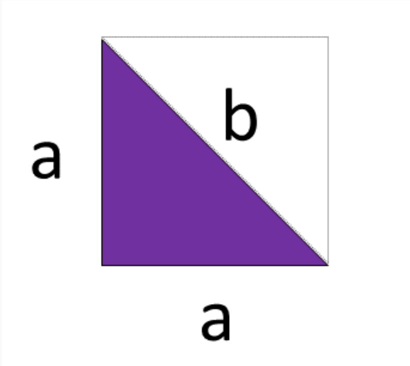
Building Bridges

Age Group
3-8



Material

The board is printed on cardboard or laminated paper.
For this activity, you could use PVC tiles. Other options include but are not limited to paper, cardboard, foam board, or you could 3D print them with PLA filament. The tiles comprise eight isosceles right triangles of the same size. You could find them as a part of Chinese Tangram.



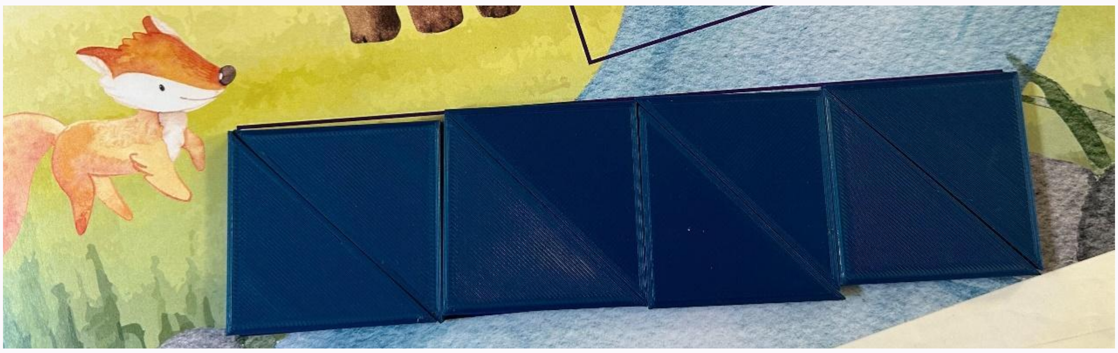
In order to obtain the triangles of appropriate size, if you use cardboard, paper, or foam board, you could cut out four squares with sides of 5cm, and then just half them through one of the diagonals. In that case, sides of triangles will measure $5cm$ (along legs of triangle marked with a) and $5\cdot\sqrt{2}\approx 7cm$ (along the hypotenuse b).
If you decide to 3D print the tiles, the dimensions of the triangles should be $5cm$ (a) and $7.07cm$ (b).

Activity

The aim is to build two rectangles with different dimensions using eight identical triangular tiles. The activity prompts children to experiment with notions of area and perimeter and to explore how different shapes can be incorporated to make bigger ones.

Solution

There is one solution for two bridges, with the same pieces in different positions, as mentioned above.



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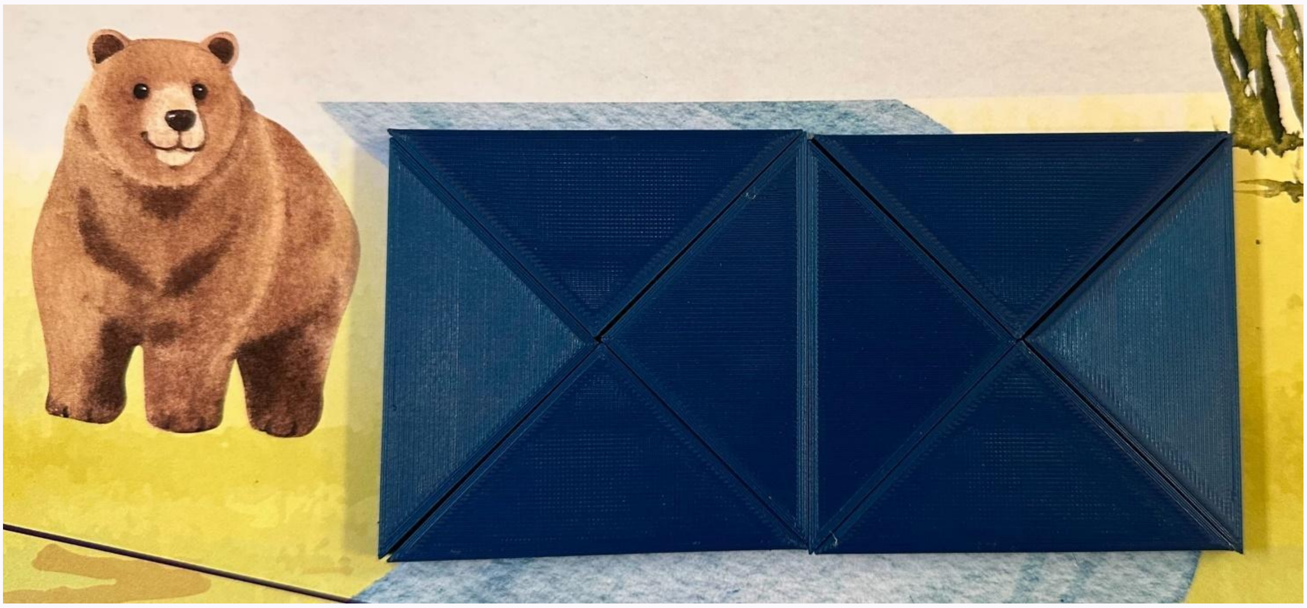
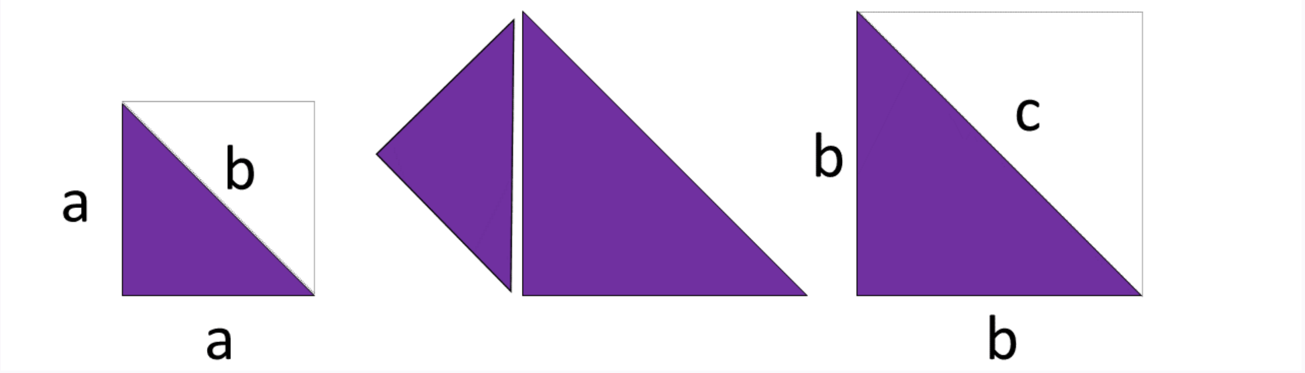
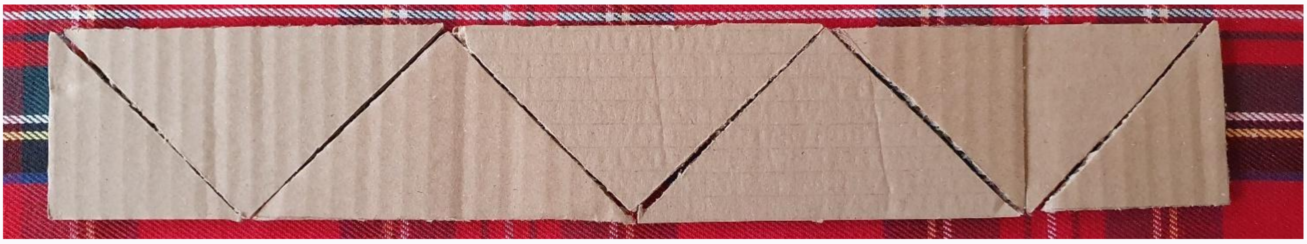


Further Explorations

Another option could be to use two different-sized sets of four isosceles right triangles to build the two bridges.



The first four triangles would have the dimensions as already explained, and the other four would have the legs of $7cm$, and then the hypotenuse would be $7 \cdot \sqrt{2} \approx 10cm$.



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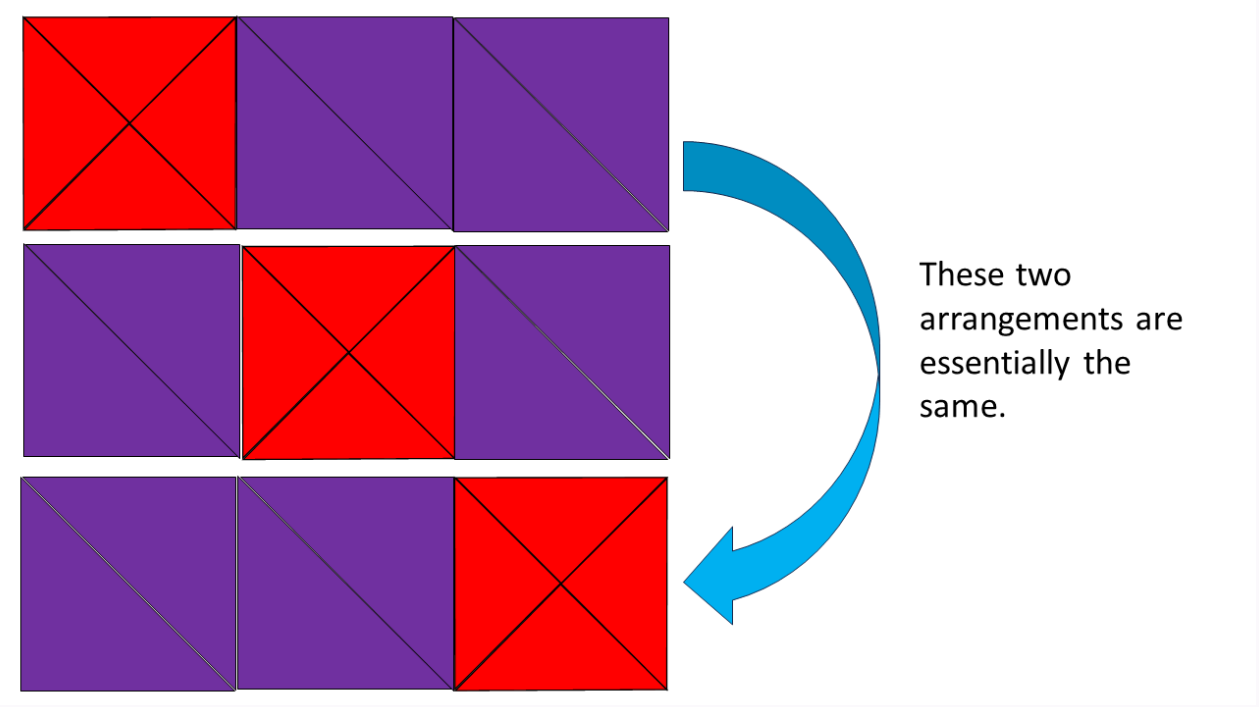
To further develop the activity, you can find the exact number of different positions of the shapes that make additional solutions. In order to do that, the triangles could be in two (or more) distinct colours or numbered accordingly (with numbers replacing colours). After listing all the possible coloured or numbered layouts, you could group them by the uniqueness of the attained arrangements.



You can level up the activity by including other shapes instead of right isosceles triangles, for instance, squares, parallelograms, or hexagons. Another activity for older kids would be to try calculating the perimeter and area of two bridges.

Mathematical Background

Geometry, the exploration of basic forms.
Spatial relationships, experimentation with rotations.
Measurement.
Combinatorics.

Transferable Skills

Experimentation
Trial and error analysis
Improving problem-solving skills.

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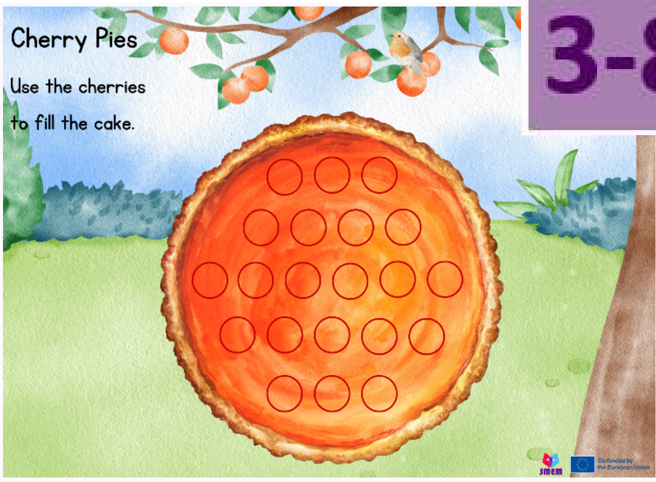
Cherry Pies

Age Group 3-8



Material

The board is printed on cardboard or laminated paper.
Cherries can be created out of wooden balls (with diameter of 30 mm), single and in groups of two, three and four connected with wooden pegs:

Single	Double	Group of 3	Group of 4	Total
3	2	2	2	21

The "cherries" can be substituted with groups of multilink cubes.

Activity

The double exhibit makes it explicitly possible that one board is always empty, and a new kid will always have a basket available to perform the activity and fill it with the groups of cherries.
If they have already found the solution, they can always look for a different one.
Furthermore, tasks can be assigned during instant workshops, when challenges can be announced like this: "You cannot use the group of 3 cherries to fill the group of three holes", or you could ask for something similar.
The older pupils could work together to find all the conceivable distributions of cherries.
It is possible to insert another similar module into the display, with holes that will also enable positioning the groups of cherries in diagonals (isometric in structure).

Solution

As said before, many solutions are possible. The idea of this exhibit is to comprehend the total number of cherries as a sum of ones, twos, threes, and fours, which means working on decomposing the number into a sum of its addends. In that regard, kids should arrange the groups of cherries only within the single rows.

Further Explorations

While acquiring the concept of numbers and computation skills, it is essential to arrange activities that will allow kids an easy shift from the number as a sum of ones to its perception as a continuous and compact quantity, that is to say, a smooth transition from counting to calculation.

Mathematical Background

Mental calculation
Composition and decomposition of numbers

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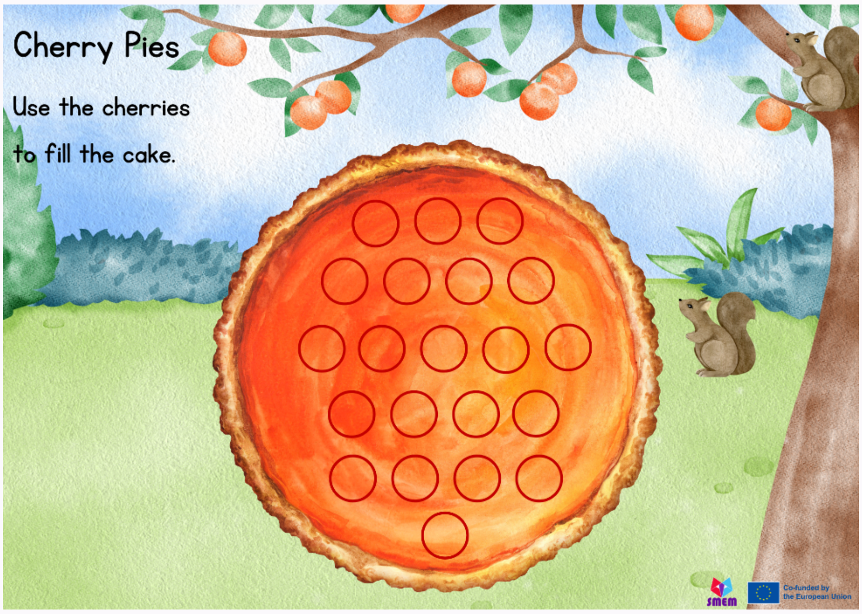
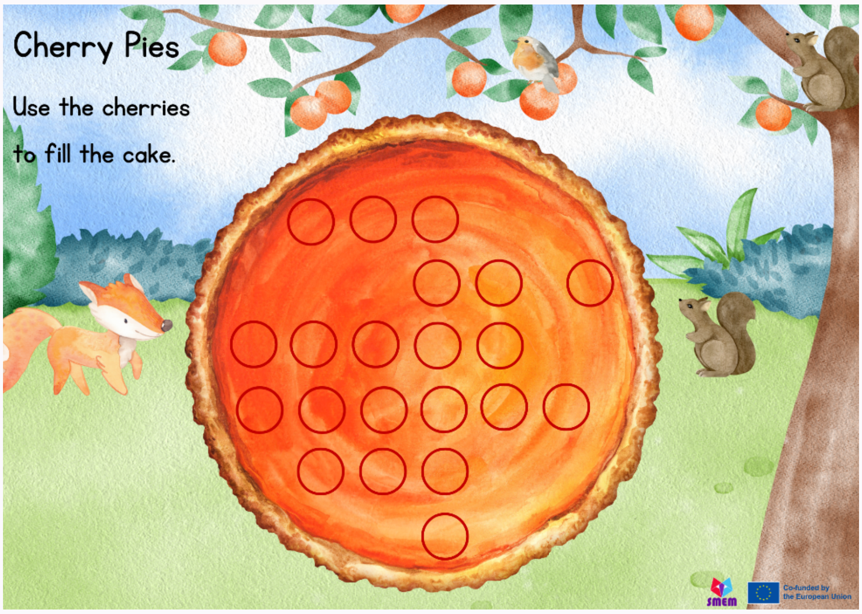
Patterns

Transferable Skills

Observation
Classification
Orientation
Trial and error analysis
Problem solving (if we give them tasks to solve)





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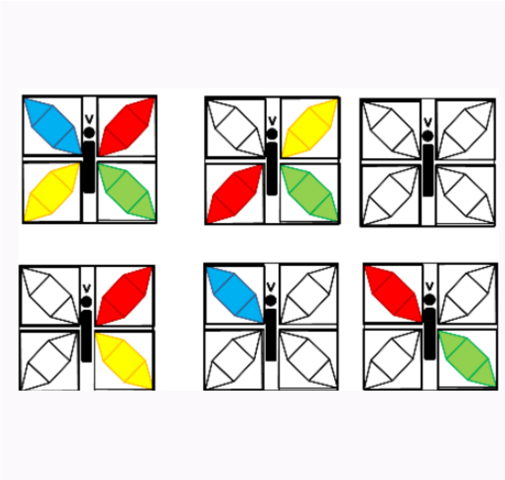
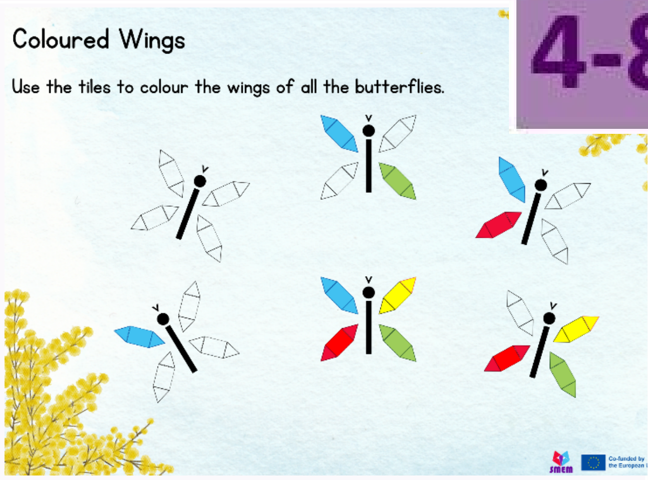
Coloured Wings

Age Group
4-8





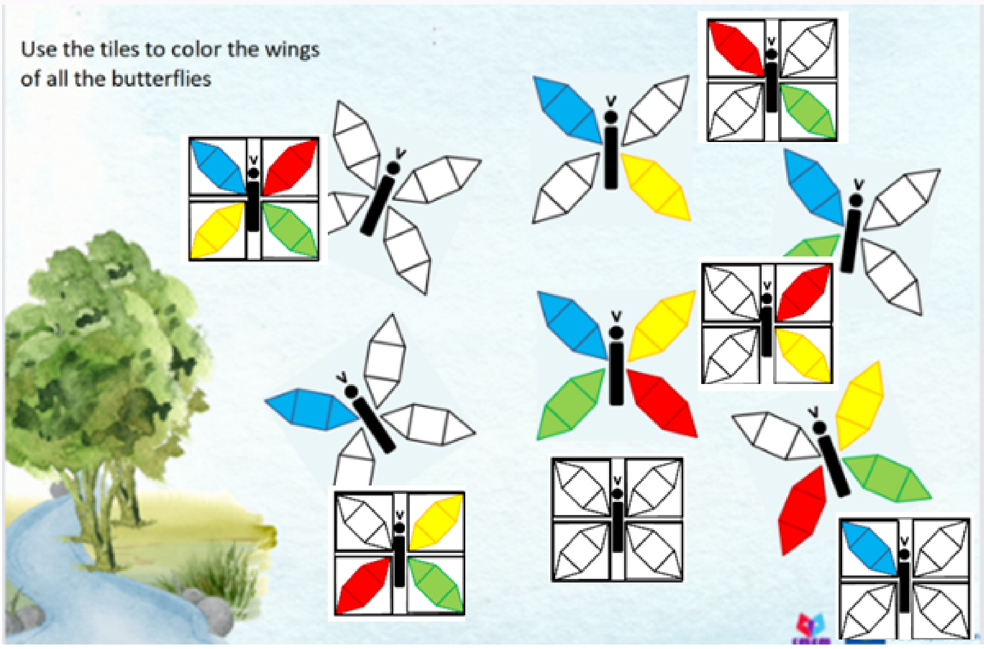
Material

The board printed on cardboard or laminated paper. Print on a transparent plastic the images to use for the activity.

Activity

The imprecision of the instructions allows various interpretations, not all aligned with a final outcome. The intention is to set a goal in the second phase. However, the idea is to complete the wings of the butterflies so that each butterfly has different colours. In this case, the colour distribution varies from butterfly to butterfly.

Solution



Further Explorations

You can easily adjust this activity to use other shapes: flowers, tiles, etc. In that regard, when preparing the new material, you could focus on various specific topics, which, depending on the age of your pupils, may or may not be known to them: combinatorics, symmetry, rotation, etc.

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Mathematical Background

Movements: translation, rotation, symmetry
Combinatorics
Patterns

Transferable Skills

Observation
Classification
Trial and error analysis
Practising hand-eye-coordination.

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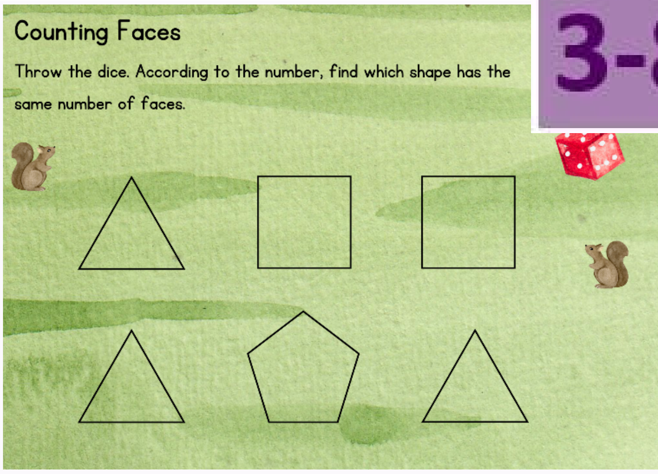
Counting Faces

Age Group
3-8



Material

The board is printed on cardboard or laminated paper. The material used to construct the different 3D shapes is PLA filament. Other options include but are not limited to paper, cardboard, and wood.

Activity

In this activity, the dice randomise the number of faces to count for each 3D shape. The shapes used in this exhibit are a cube, a tetrahedron, an octahedron, a dodecahedron, and an icosahedron. The rationale behind this is to guide children into discovering how we can represent 2D shapes in 3D form and engage them in calculating the number of faces as an introduction to geometry.

Solution

Based on the nature of the activity, the solution depends on the number of faces of each object.

Further Explorations

You could use Platonic solids to discover the properties of 3D shapes (length, width, depth) and count their edges and vertices. Extension to area measuring by providing larger 3D shapes containing the smaller ones is also possible.

Mathematical Background

Geometry
Mental arithmetic
Addition
Measurements

Transferable Skills

Observation and experimentation.

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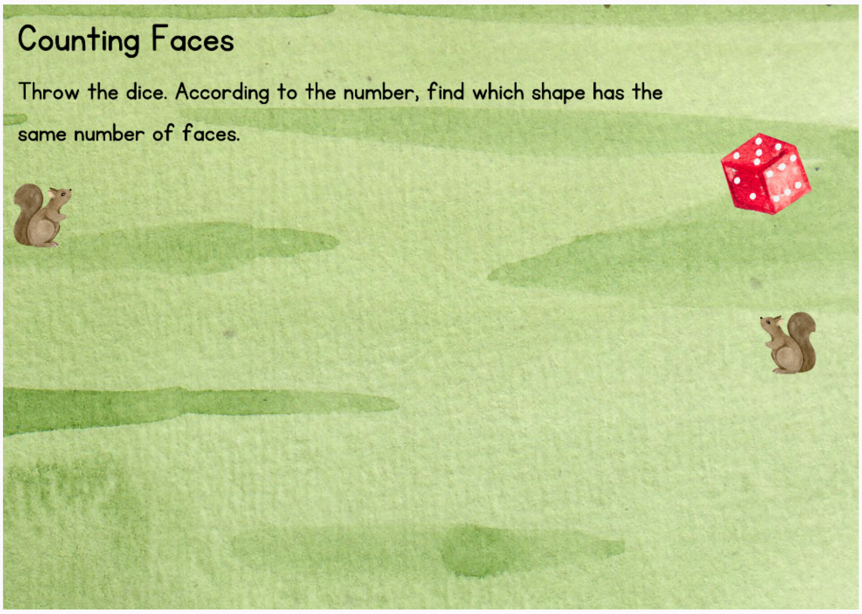

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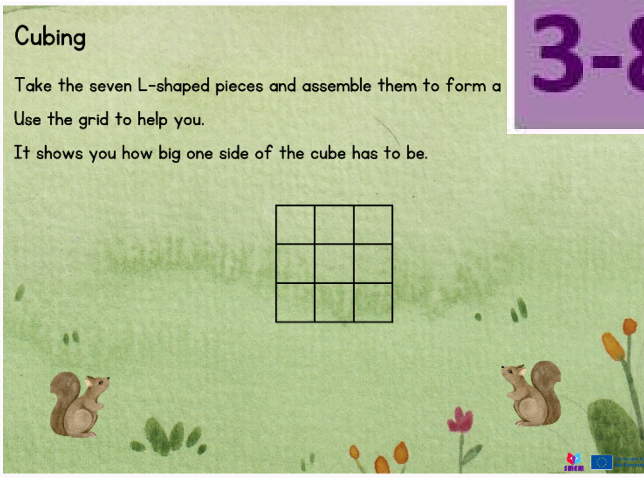
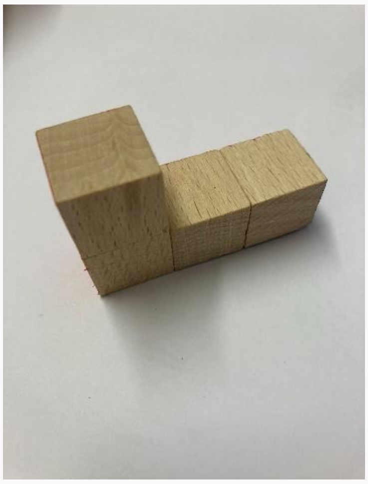
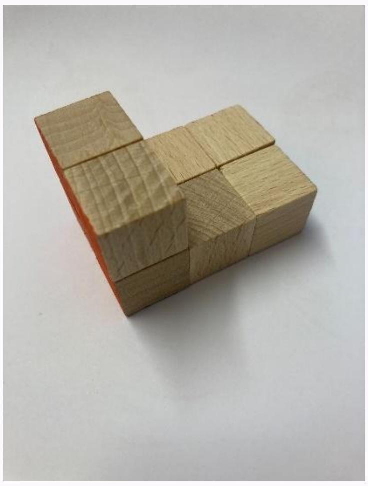
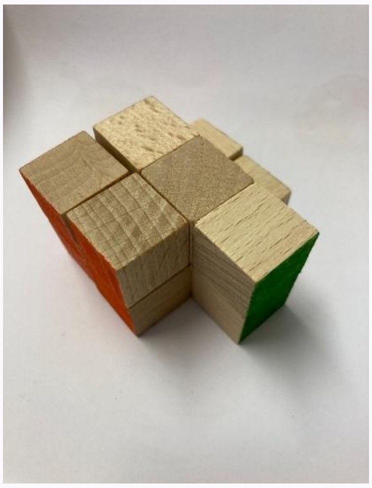
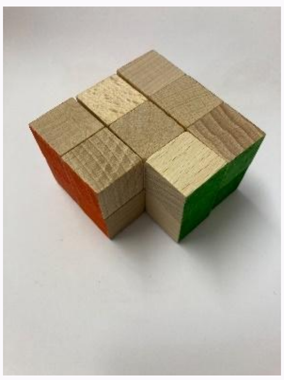
Cubing

Age Group
3-8



Material

The board is printed on cardboard or laminated paper.
Twenty-seven (wooden) cubes with a side length of 2 cm. (Wood) glue.
Six different colours for painting the sides of the cube. Glue four by four pieces together to form an L shape (see picture). Do this until only three cubes are left. Then, glue these leftover cubes together to form a small L shape. Assemble a cube (3x3x3) and paint each side of the cube in a different colour.
We would suggest to assemble the cube in the following way:


Activity

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The activity can be divided into several questions, depending on the kids' age:

1. How many pieces are there?
2. What shape do they have?
3. How many small cubes are there in each piece?
4. How many small cubes in total?
5. How many colours are there? Which colours do you see?

Now, the kids should try to assemble a large cube. Each side of the large cube consists of nine small cubes (3x3). As a helpful hint, use the board grid to arrange the shapes, as it indicates how big one side of the cube has to be. Find all the pieces with yellow colour and make a yellow square.

Solution

The easiest way to start is by collecting the pieces that show one of the colours. Then, create a square out of them. After assembling the "bottom" square, it is easy to find the solution, respecting that each side of the large cube should be in one colour.

The cube has only one unique solution when each side is in only one colour. The cube has many more solutions when mixing colours on each side of the cube.

Further Explorations

You could not only build a cube out of the pieces but other forms using not all of the Ls.

There are several ways to divide a 3x3x3 cube into smaller pieces. In this exhibit the cube was divided into 3 Ls. The most common way is the soma cube. There are also 7 pieces made out of 3 to 4 cubes, but they are all different. One could build a Soma-Cube with older children and try to find its solution.

Mathematical topics

Combinatorics

Patterns

Relationship between plane and space (2D and 3D objects)

Transferable skills

Spatial recognition

Practising fine motor skills.

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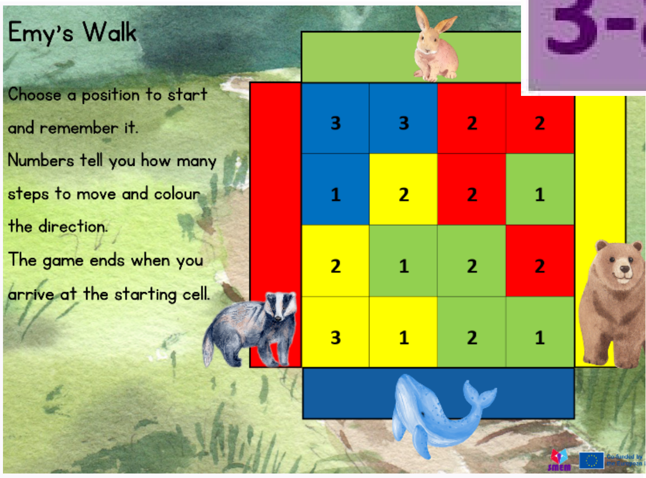
Emy's Walk

Age Group
3-8



Material

The board is printed on cardboard or laminated paper. A token to move across the board.

Activity

The activity is quite simple, although to start, especially with younger kids, you will have to repeat the instructions.

Once you break the ice, the game surprises and pleases.

We suggest following observations:

Will we manage to go through every square of the board?

How many steps will it take us?

Solution

As mentioned above, it is possible to make the full circle starting from any square on the board. As in many other exhibits, more mathematics is applied in designing and building them than in solving the challenges presented to us.

Further Explorations

It is pretty straightforward to design new and even bigger labyrinths and create a personal collection.

You could use arrows instead of colours in each box to specify in which direction to move from any board field.

Mathematical Topics

Counting

Orientation

Transferable Skills

Observation

Trial and error

Following instructions

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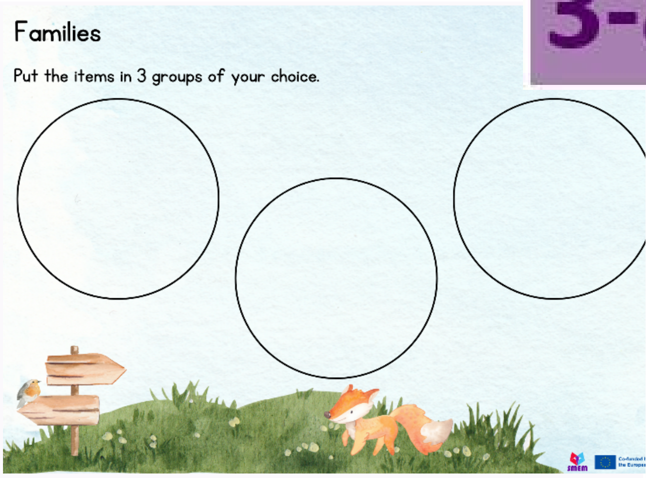
Families

Age Group
3-8



Material

The board is printed on cardboard or laminated paper.
The material used to construct the different 3D shapes is PLA filament. Other options include but are not limited to paper, cardboard, and wood or Logic Block tiles.

Activity

During the activity, children will compare and classify the shapes into three groups. They can use whatever criteria they want. Younger kids will work with the tokens shaped like animals of different sizes and colours. Older kids will use geometrically shaped tokens such as triangles, circles, and squares. They could use both 2D and 3D shapes. The idea is to enable the learners to classify the items into three distinct groups based on the chosen rules. They could use differences or similarities as a guiding criterion.

Solution

There can be multiple solutions to the groupings depending on the required difficulty and established guidelines.

Further Explorations

If you use the animal figures during this activity, you could extend it by classifying other real-life objects as similar or different from each other based on their colour, shape, and size.
For the activity with geometrical shapes, the extension could include measuring the shapes and calculating their area and perimeter.

Mathematical Topics

Geometry
Measurement
Observation
Patterns

Transferable Skills

Problem-solving
Experimentation
Trial and error analysis

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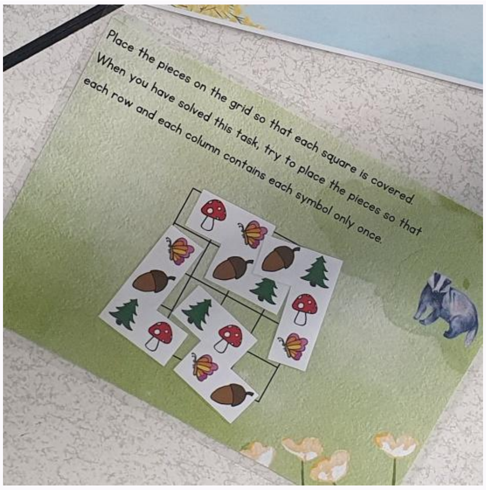
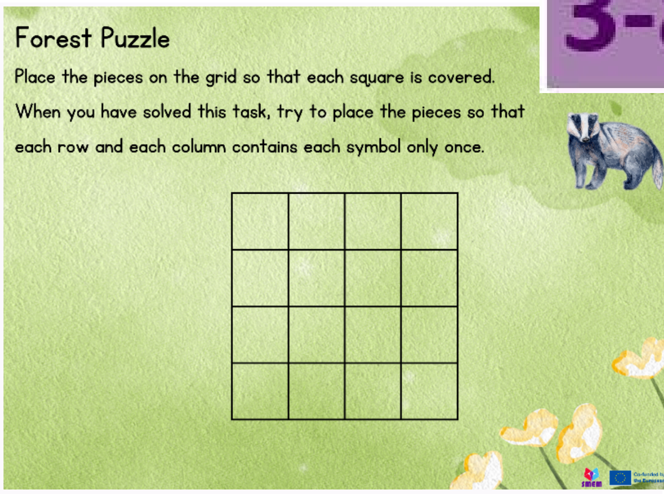
Forest Puzzle

Age Group
3-8





Material

The board is printed on cardboard or laminated paper. Printed puzzle pieces. Cut them out and laminate them after printing.

Activity

Depending on the children's age, you should start the activity by asking the following questions:

- What images do you see?
- How many puzzle pieces are there?
- How often does each image appear?
- Can you position the pieces in a way that they form a square?

The last question is the hardest and is also the main task of the activity: Can you arrange the puzzle pieces so that each image appears only once per row and once per column?

Solution

There is only one unique solution.

Further Explorations

What would a possible 3x3 square look like? Draw a 3x3 grid and try to fill it with nine tokens of three different colours or images. The governing rule is that you cannot repeat colours or images within a single row or column - they may appear only once. Then, cut the square grid into three pieces to make a custom puzzle.

You could also try to find a solution for a 5x5 grid.

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Mathematical Topics

Combinatorics
Patterns
Relationship between plane and space (2D and 3D objects)

Transferable Skills

Spatial recognition

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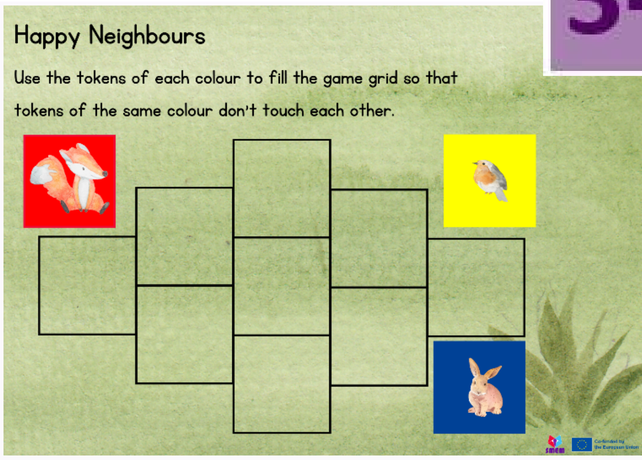
Age Group

3-8

Happy Neighbours



Material

The board is printed on cardboard or laminated paper. Tokens created by printing in colour on cardboard and then cutting out the shapes. Alternatively, 9 tokens in 3 different colours, 27 tokens in total.

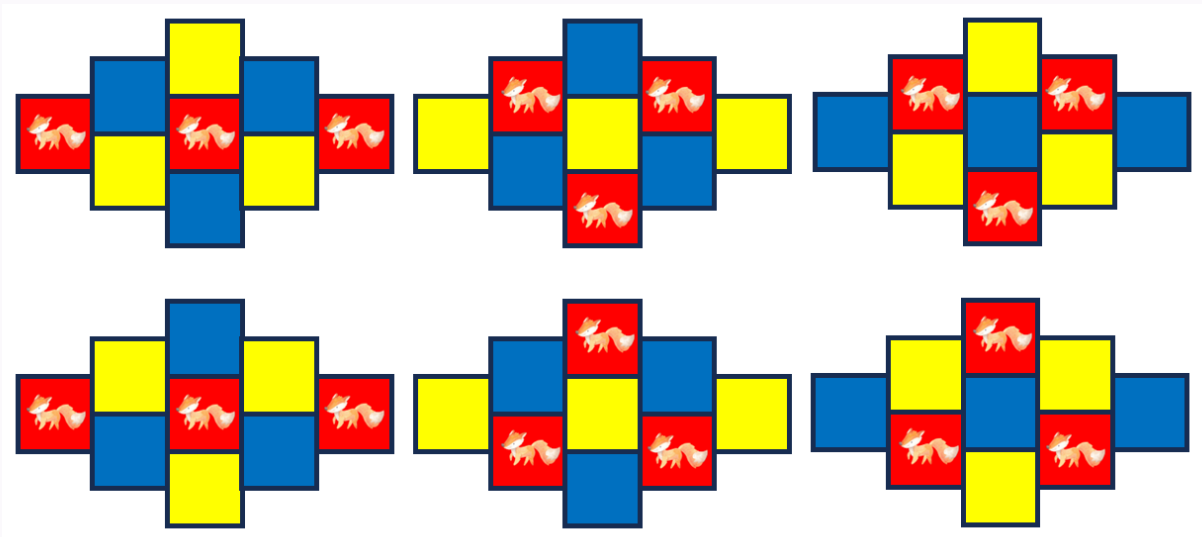
Activity

This activity could be done by a single student or by working in pairs or even groups of three kids. One kid tries to solve the puzzle by himself/herself. Two kids take turns in placing tokens of all 3 colours. Three kids are placing one colour each.

The aim is to fill the grid with tokens in such a way that there are no two of the same colours next to each other, in other words, the animal neighbours are happy when having some other types of animals as their neighbours.

Solution

There is one unique solution to the original problem which, by using symmetry, generates six different placements of the tokens that follow the rule.



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Further Explorations

The modification of this activity is different in reference to student's age:
(3+) By using 9 tokens (3 of each colour), the teacher places the first token in the central field of the grid, and leaves the rest for kids
(5+) By using 9 tokens (3 of each colour), the teacher covers all 9 fields in a way that the general rule is not fulfilled. Kids should modify the order of tokens by following the rule.
(7+) Introduce 27 tokens (9 of each colour). Cover the grid with 9 tokens of the same colour. The kids should replace the tokens by following the rule. How many different colour tokens are at least needed?

Mathematical Topics

Geometry
Graph Theory

Transferable Skills

Hand-eye coordination (placing tokens on the board)
Strategic thinking (placing tokens in accordance with established rules)

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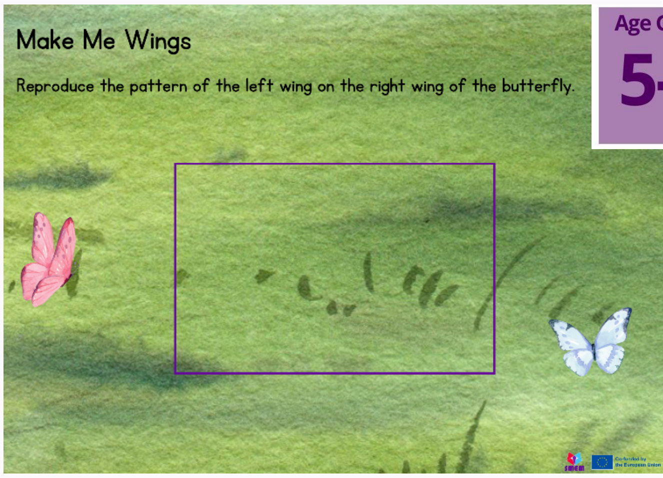
Make Me Wings



Age Group
5-8

Material

3D printed board or game grid are printed on paper/cardboard.
28 different geometrical shapes printed with a 3D printer or made from cardboard. Alternatively, you could use Pattern Blocks.
Two model cards to print.

Activity

The activity consists of two parts:
Can you reproduce the card design on the correct butterfly wing?
Can you make the symmetrical paving on the other wing with the leftover shapes?

Solution

The first task is to reproduce the card model on the butterfly's wing by recognizing the geometric shapes used and positioning them in the proper place on the butterfly's left wing.
Next, kids should make a symmetrical model of the left wing on the butterfly's right wing. To complete the task, they should recognize which shapes to use. The shapes should be arranged on the opposite wing symmetrically, with the butterfly's body as the axis of symmetry.

Further Explorations

This activity enables kids to familiarise themself with simple geometric forms and recognize less commonly used geometric shapes, such as trapezoids of different sizes and properties. It also allows them to reproduce a diagram and deduce its symmetrical shape.
To go further, it would be interesting to literally repeat the symmetrical shape of the model provided without replicating the pattern. In other words, first position the tiles on the right wing, then make the symmetrical shape on the left wing and use the model card as a correction.
This activity also allows you to free your imagination by creating the pattern of your choice with the geometric tiles provided and then replicating the activity: making the symmetrical pattern on the other wing of the butterfly.

Mathematical Topics

Geometry
Symmetry
Spatial relationships

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Transferable Skills

Name and identify shapes
Reproduce a paving pattern using different shapes
Replicate the paving using axial symmetry
Hand-eye coordination (building the pattern)

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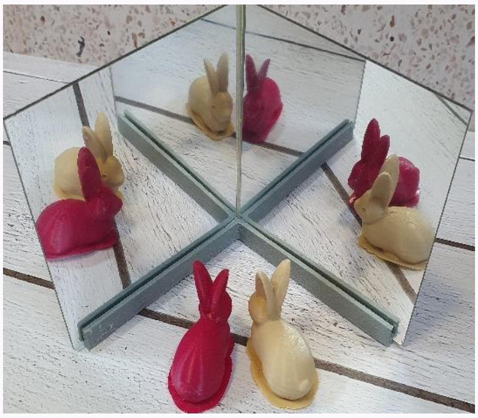
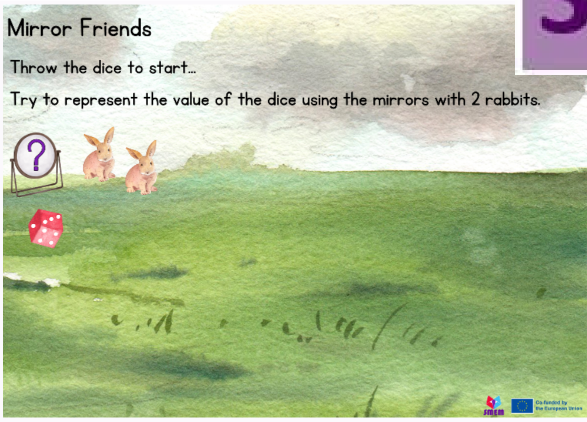
Age Group
3-8

Mirror Friends





Material

The board printed on cardboard or laminated paper.
One die or two dice.
Two or four tokens (chess pawns, little figures of animals, etc.).
One flat mirror with a side of 10 or 15 cm.
One double mirror, with the same side and, with a fixed angle of 90º or 120º.
We don't recommend using glass mirrors. You can find methacrylate mirrors 3-4 mm thick or PVC mirrors glued onto a rigid support (wood) at a reasonable price.

Activity

Two difficulty levels are available for this activity.
For the simpler version, you need one die, two rabbits, and two mirrors (one plain mirror and two mirrors with a 120º angle in between).
For the more difficult one, you need two dice, three rabbits, and two mirrors (one plain mirror and two mirrors with a 90º angle in between).
In both cases, the goal is the same: you have to make as many rabbits (or any other object you decide to use) as indicated by the throw of the dice by using the mirrors' properties.
If the group of kids is doing this activity, an additional task could be to find alternative ways of representing the throw(s) of the dice.

Solution

Roll the die and count the score.
For instance, to get number four, you could put two rabbits in front of the flat mirror or one rabbit in front of the double mirror. For number six, you could put one rabbit in front of the sole mirror (you will get two) and one rabbit in front of the double mirror (you will get four).
If you work with three rabbits, you should catch the impossibility of obtaining the number eleven. It is an interesting example as it gives rise to a conversation. In some cases, it could induce frustration. To resolve this, we suggest using four rabbits at the beginning. It is an option that increases the number of alternative combinations.

Further Explorations

It is possible to add another double mirror, with a 120º angle, so that by putting one rabbit in the middle, three rabbits appear.

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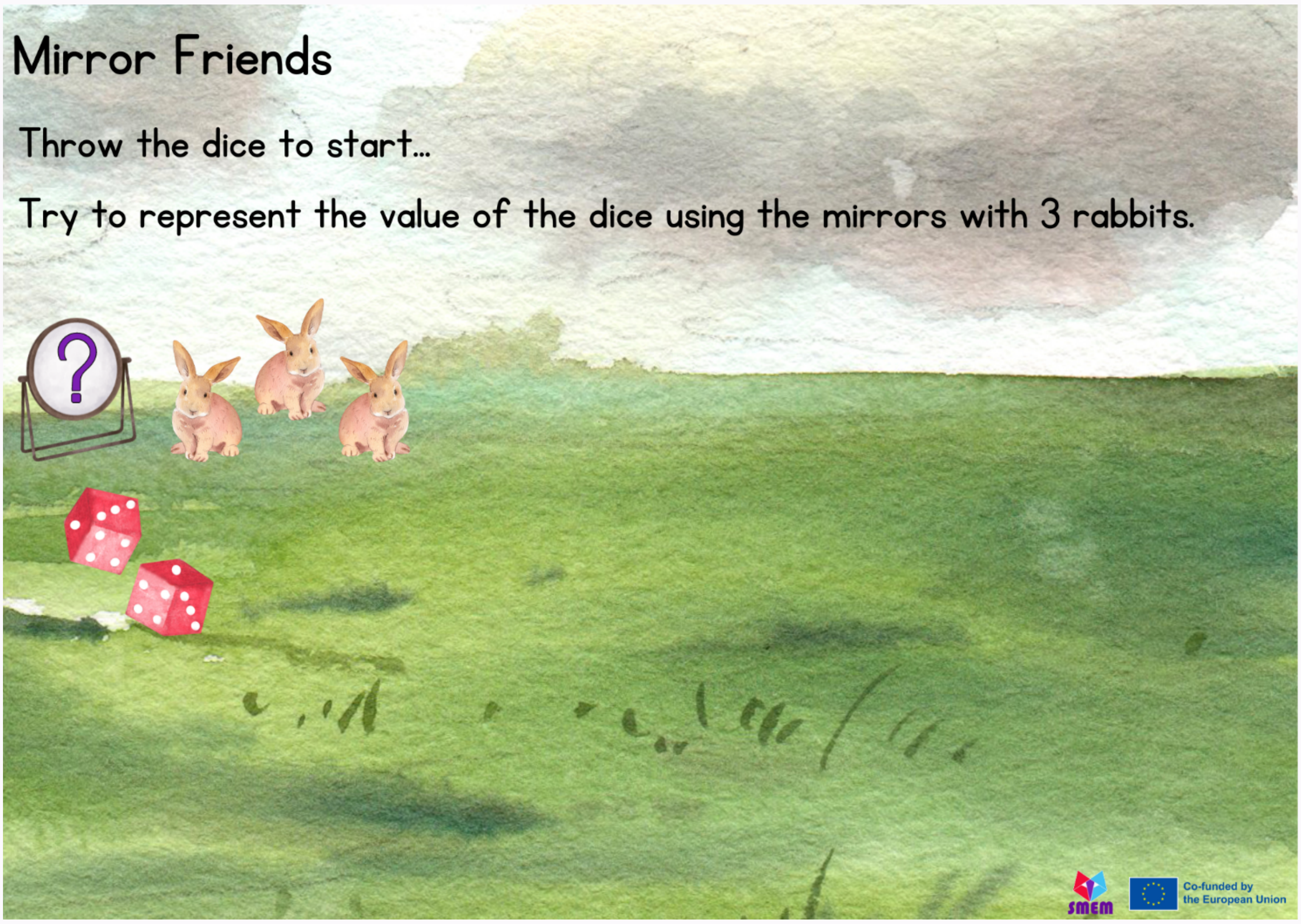
A further exploration would be introducing the number zero (represented by a box with a canvas lid containing a cut for enabling the rabbits to disappear) and asking to use all the rabbits available to depict any number on the dice.

Mathematical Topics

Counting
Mental calculations (approach to addition and multiplication)
Number composition and decomposition
Zero as neutral element of the sum (if we introduce the box)

Transferable Skills

Observation
Trial and error analysis
Problem solving
Approach to computational thinking (if we introduce the box)



Seaside Selfies

Age Group
3-8

Material

The board is printed on cardboard or laminated paper.
The material used for the phone frame can be foam board or cardboard. The dimensions of the inner frame are 9 cm x 16 cm.

Activity

In this activity, kids use a camera lens to take the pictures depicted on the second board. The idea is to experiment with angles, distance, and measurements, as well as spatial recognition and awareness of the space, positioning, and order of different objects in a picture.

Solution

The angle and distance (zoom in and out) of the frame match the pictures provided on the board with the instructions.

Further Explorations

The angle and distance (zoom in and out) for positioning the phone frame should enable matching the pictures "taken" with the ones depicted on the board with the instructions. As an additional activity, you could introduce phone frames with different ratios (3:4, 1:1) and prepare mixed ratio photos as a guiding board.

Mathematical Topics

Geometry
Spatial relationships
Angles
Position
Order
Space

Transferable Skills

Problem-solving
Experimentation
Trial and error analysis

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Try to take the same picture with the phone camera. Remember to zoom in or out.

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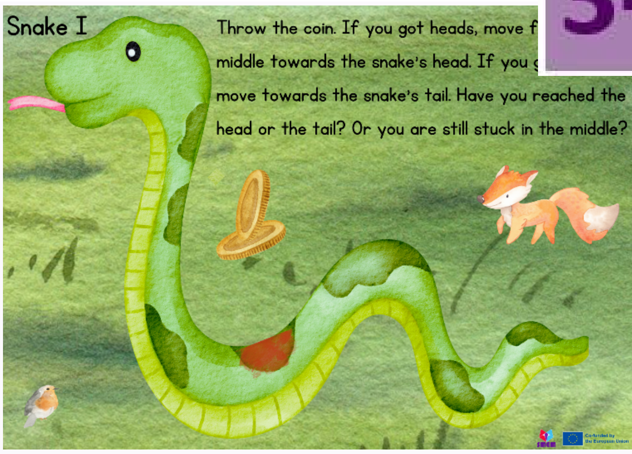
Snake I (exploration game with coins)

Age Group
3-8



Material (if applicable)

The board printed on cardboard or laminated paper; optionally black outline of the snake printed in transparent PVC.
Two real-life coins
One token to move along the snake

Activity

The teacher asks students to find a partner and places the token on the red patch on the snake's back. The children each take turns tossing their coin and move the token accordingly - towards the tail or the head of the snake by using the green patches. Three green patches are available between the red patch and the head, and the same number of green patches is available between the red patch and the tail.
The activity is different in reference to student's age:
(3+) Have you reached the snake's head or tail?
(5+) Modify the game: one student only throws the coin, the other only moves the token accordingly. If you use additional light green fields for moving the token, is it easier or harder to reach head or tail?
(7+) How many throws did you need to reach the head or the tail? Record throws with lines in order to be able to answer. What is happening with the token if we increase the number of rolls?

Solution

The game is based on the probability of throwing a head/tail being 50% in a single toss (so it is equally probable the next move will be toward the head or the tail). With more tosses, it starts getting tough to reach the end, as the token will be moving around the middle red patch. But, as there are only three fields between the start and the finish (head or tail), this game will end eventually.

Further Explorations

If you want to make it harder to finish and easier to get the feeling for probability of a coin, introduce more fields towards head/tail.

Mathematical Topics

Probability
Statistics

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Transferable Skills

Hand-eye coordination (throwing coins, dice; moving pieces along the snake)
Identify coin sides (heads, tails)
Identify direction (towards head or tail of the snake)
Notice that if the number of throws is getting bigger, it gets harder to finish the game

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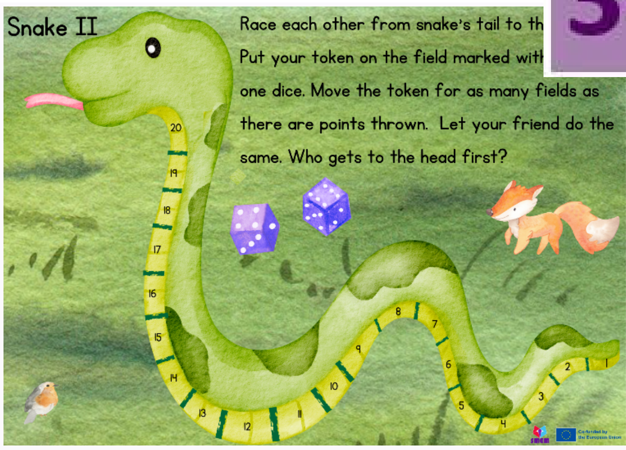
Age Group
3-8

Snake II (race game with dice)



Material (if applicable)

The board printed on cardboard or laminated paper; optionally black outline of the snake printed in transparent PVC.
Two dice
Two different tokens to move along the snake

Activity

The teacher asks students to find a partner. The tokens are placed on the snake's tail (optionally, that field is marked with 1). The children each take turns rolling their dice and move their tokens accordingly. The activity is different in reference to the student's age:
(3+) Who won the race?
(5+) Modify the game: by using counting in twos: if you get 3 when rolling the dice, move the token further for 3*2 fields by counting 2-4-6. Who won the race?
(7+) Modify the game: by using counting in threes (fives): if you get 2 when rolling the dice, move the token further for 2*3 (2*5) fields by counting 3-6 (5-10). Who won the race?

Solution

The game is aimed at practicing counting, recognizing numbers and patterns.

Further Explorations

For this activity with older children, we suggest counting in twos, threes, and fives, which would be a gentle introduction to multiplying numbers. As the board with 20 fields would quickly be outplayed, in this case, it is better to use the board with 40 fields.

Mathematical Topics

Arithmetic
Counting
Observation

Transferable Skills

Hand-eye coordination (throwing coins, dice; moving pieces along the snake)
Identify numbers (with dice throws on the board)
Identify direction (towards head or tail of the snake)
Comparing numbers (it is better to get bigger numbers when rolling dice in order to finish first)

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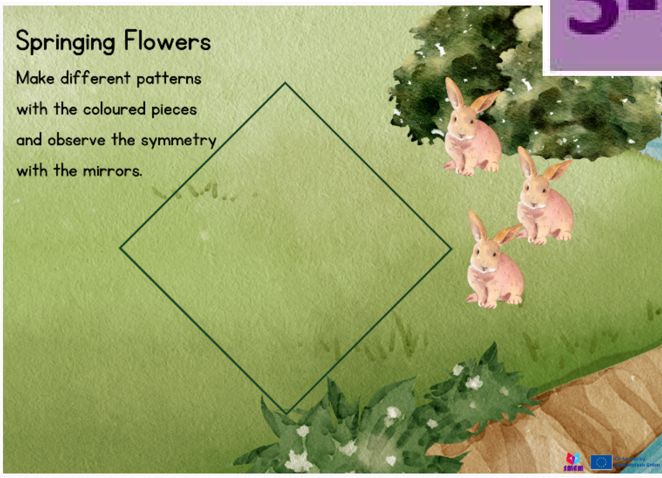
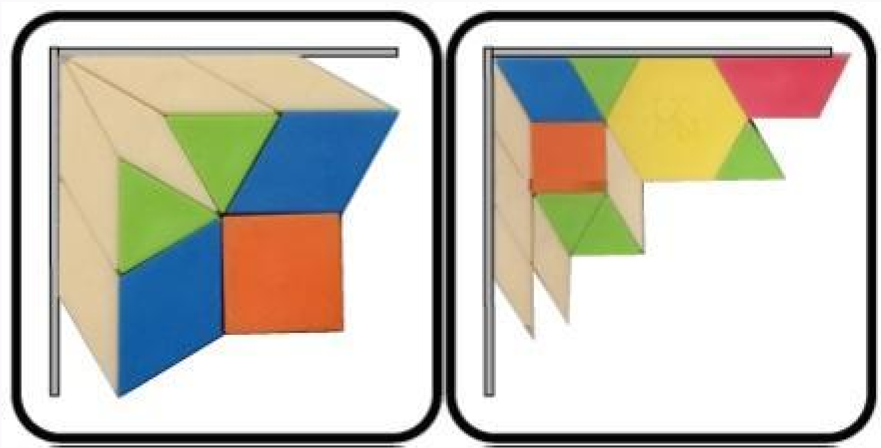
Springing Flowers

Age Group
3-8



Material

The board printed on cardboard or laminated paper. 3D printed support for mirrors and shapes, two 15cm x 15cm mirrors 3mm thick, various small "attrimaths"-type geometric shapes, six pattern paving cards to print.



Activity

The activity is divided into following parts:
Can you reproduce the different layouts of the pattern cards?
Can you imagine a tiling design using the geometric shapes provided?
Can you see the pattern in the mirrors?
Can you find the different axes of symmetry?

Solution

To begin with, the students should replicate the paving of the pattern cards. To do this, they should identify the axes of symmetry of the design on the card, which correspond to the mirrors on the manipulative device. Then, the kids should find the correct geometric shapes and how many tiles they need. After building the design, they should check whether it is the same as the one on the card.
Next, the children create their paving using the mirrors. The aim is to tile the surface without gaps or overlapping shapes by matching and recognizing each geometric shape depicted in the pattern card.

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Then, they could deduce the area to be tiled with the desired pattern or to see it in its entirety. Finally, identify the axes of symmetry in a created paving or geometric shape.

Further Explorations

Firstly, while working with the model cards, it is interesting to discover the geometric shapes utilised in the proposed paving and the number of tiles used. The model card indicates the complete paving, meaning the paved surface and the mirrored symmetries. The activity could involve kids figuring out the area they need to tile so that the paving is identical to the model map.
Next, using the geometric shapes supplied, create your paving and visualise all the symmetries with the mirrors.
To take the activity one step further, you can compare shapes to identify the ones with identical sides. Next, pay attention to the angles. Try to figure out the sum of which is the same as the angle of another geometric shape, allowing you to alternate patterns. These different activities, which use observation and manipulation, help students learn about geometric shapes and axes of symmetry.

Mathematical Topics

Geometry
Spatial relationships

Transferable Skills

Name and identify shapes
Work on axial symmetry
Compose tiling blocks

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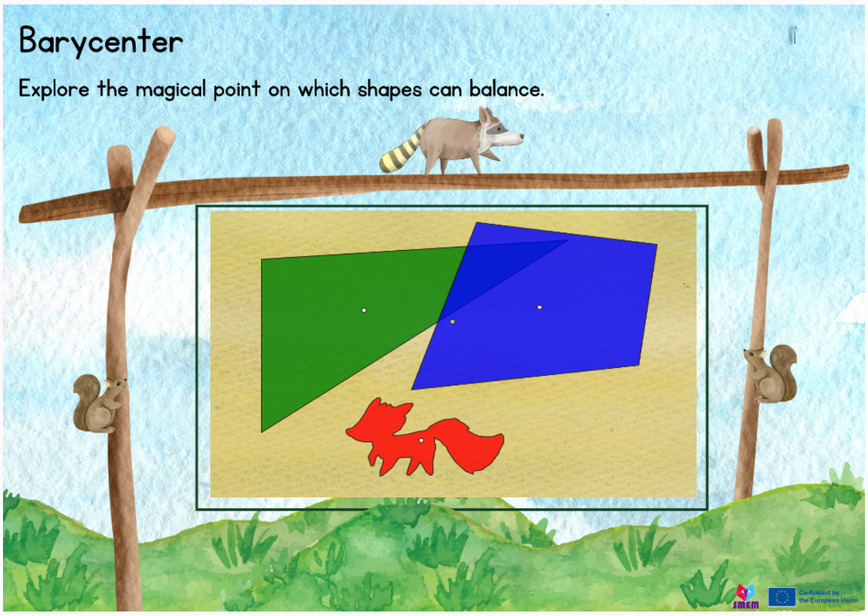
Barycentre

Age Group
6-8



Material

The board is printed on cardboard or laminated paper and a tablet or a computer.
The application allows you to draw up to three shapes. You can do it freehand (dragging the cursor/finger) or joining straight segments (tapping on different points). You can move or modify the shapes if necessary. Some sample shapes are available to be selected.
For each shape, its barycentre is automatically calculated and displayed. Additionally, the application displays the barycentre of all the shapes combined.
The extended version of the program includes a print button that allows the user to download a PDF with the drawn shapes (independently and combined in one image). The user can then print the pdf file, glue the paper on cardboard, and cut the figures to obtain a physical object with the drawn shapes. You could use them to construct a crib mobile toy. In this case, you need a printer, paper, cardboard, scissors, stick glue, thick string, and a paper clip.

Activity

This exhibit dives deeper into the barycentre (centre of mass) topic. The application is a tool for exploring different phenomena rather than a puzzle with a solution. The educator can engage kids with a set of activities or challenges.
If the transparent pieces from the PR1 exhibit are available, you could place one of the shapes on top of the screen, follow the silhouette with your finger or stylus, and locate the barycentre. Use it to balance the shape on the tip of a pencil.

Further Explorations

In addition to finding the barycentre of one shape, the program can compute the combined barycentre of two and even three different shapes. Here are some questions that you could use to explore these properties:

- Draw a shape that has its barycentre outside the shape. How can you balance such a shape?
- Draw a triangle. Where is its barycentre? How can you find it geometrically?
- Draw a quadrilateral. You can split it into two triangles. How can you find the barycentre of the quadrilateral with the help of the barycentre of the two triangles? (Tip: Draw a quadrilateral as a single shape or composed of two triangles obtained by splitting the

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quadrilateral along one diagonal. The combined barycentre of the two triangles is the same as the barycentre of the quadrilateral.)

* Take one of the shapes that balance horizontally on a wall (see “looking for an equilibrium” exhibit. Draw it on the tablet with the line that splits it into two parts, one for each side of the wall (just like you already divided the quadrilateral in the previous example). Compare the barycentre of the parts with the one of the shapes.

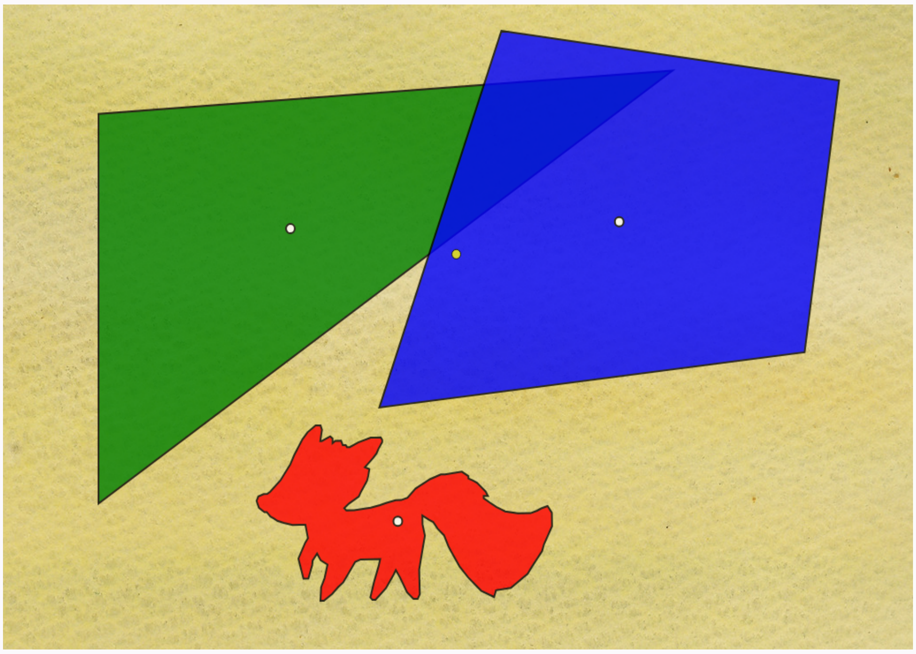
A more advanced challenge is to list as many methods as possible to find the barycentre of a planar shape.

Mathematical Topics

Barycentre
Averages (arithmetic mean, weighted mean)
Lever principle

Transferable Skills

Exploring mathematical properties
Following a procedure to find a solution
Making conjectures to explain a phenomenon
Practicing fine motor skills (if building the physical toy)



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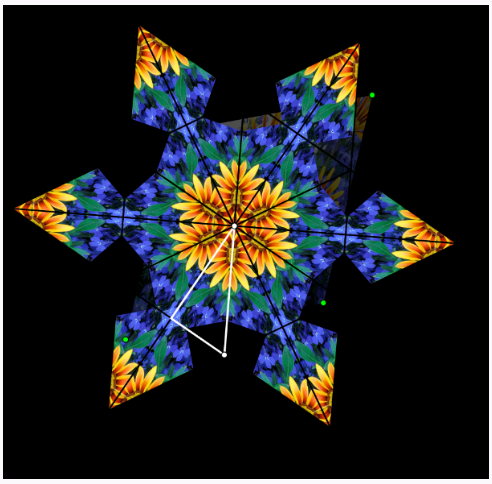
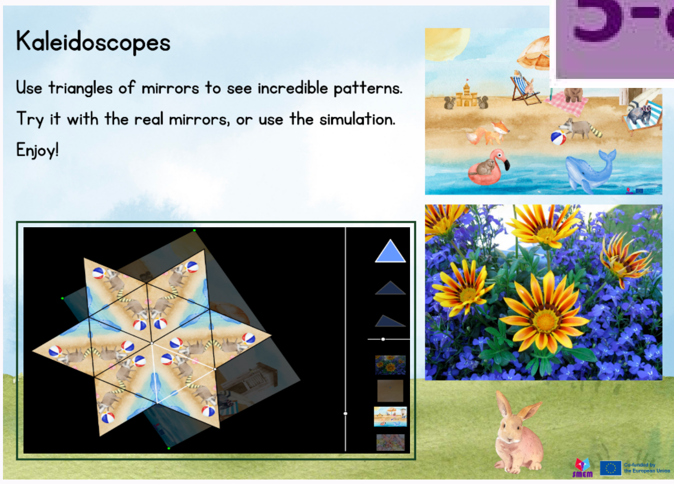
Kaleidoscopes

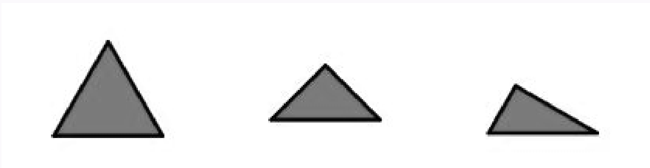
Age Group
5-8





Material

Hybrid exhibit. Compare the physical and the virtual versions of the same kaleidoscopes. Kaleidoscopes are sets of two or more mirrors that reflect each other. Here we consider kaleidoscopes made of three mirrors on the sides of special triangles with angles (60,60,60), (90, 45,45), and (90, 60, 30). The reflections on these kaleidoscopes pave the plane with copies of whatever object we place inside.



The physical version consists of kaleidoscopes built with actual mirrors (plastic ones, for safety reasons) making up the lateral faces of a prism, with the mirror facing inside. To see the reflections, you need to look from a side into the prism.
On the tablet, you could choose an image to put "on the table", and the application will simulate its reflections on the mirrors. A slider allows one to generate the reflections step by step instead of at once, making it easier to observe and analyse. A comparison of the physical and the virtual objects enhances the comprehension of the phenomenon.

Activity

- If using the physical kaleidoscopes:
  Take a coin and place it near a vertex of the kaleidoscope. How many copies of it do you see? Make a list of angles and number of copies.
  Place the coin a tad further from the vertices and look at the reflections around each vertex. Can you count the number of copies? Or could you list the copies somehow?
- If using the virtual kaleidoscopes:
  Use the application with an image of a coin and make it turn for the coin to get closer to the three vertices. Repeat virtually the previous experiences.
  Use the slider to go from just a few copies to many copies. Try to explain how many copies are in each "generation." Try to deduce how they emerge.

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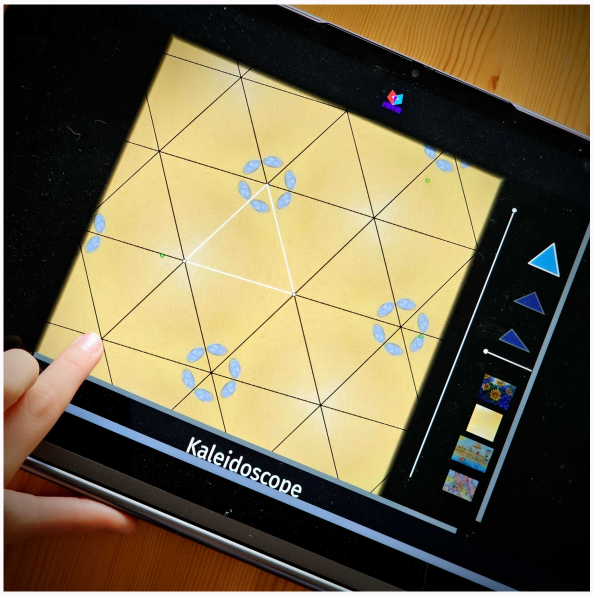
Solution

The number of copies around a corner of angle α is 360°/α. So, for a 30° angle, twelve copies; for a 90° angle, four copies, etc.

In the application, the initial generation has as many copies as 360°/α where α is the angle in the centre. Every generation unfolds the triangles in the previous generation, so every generation appends the same number of copies.



Further Explorations

Leaving the kaleidoscope spinning is by itself a pleasant experience. Younger children can try to find the animals on the beach, or play with geometric motifs.

Some questions to explore further:

Compare this exhibit to the "Mirrors" exhibit. What happens when the angles are not "nice" angles, like 30°, 45°, 60°, 90°?

For older children that can read: Write a (non-symmetrical) word inside the kaleidoscope. How many of the copies can you read?

Mathematical Topics

Geometry
Wallpapers, tiling
Symmetry
Reflections

Transferable Skills

Notion of infinity
Exploring geometric properties
Intuitive understanding of symmetry and angles
Enjoying the beauty of this part of mathematics

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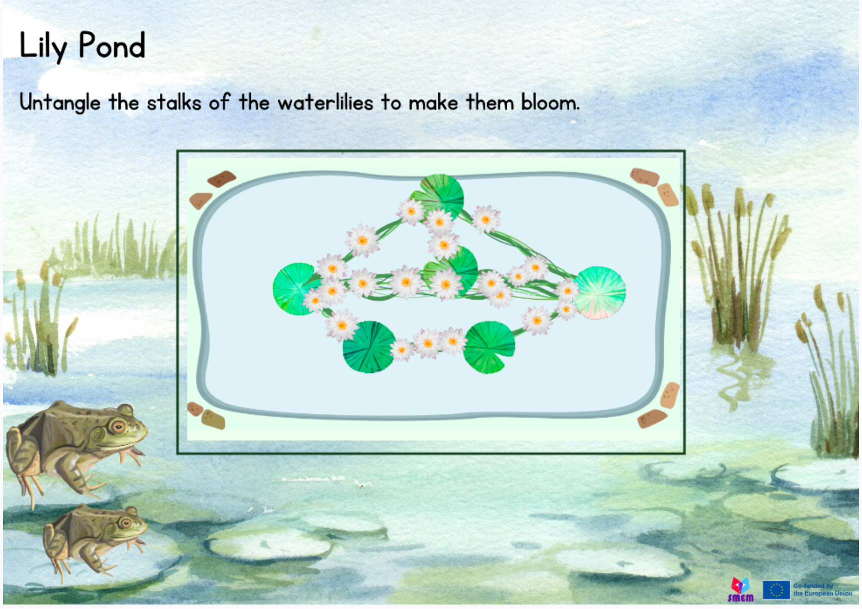
Lily Pond

Age Group
3-6



Material

The board printed on cardboard or laminated paper. Tablet or a computer.

Activity

The application shows several lilies in a pond. Some of them want to be joined by a plant stem. You can identify those by a line connecting these flowers. However, the stems are floating on the pond surface and cannot cross each other. Your task is to arrange the lilies to make their stems able to grow, which means no crossing should be present.
When you solve the challenge, you can reload the page, and the application will generate a new puzzle. The application is programmed to make the puzzle always solvable.

Solution

The challenge is to find a planar embedding of a graph. Although there are algorithms that solve this problem, it is often easier to follow some heuristic rules:

- Position first the nodes with more edges.
- Group triangles and nodes into untangled clusters and continue growing the graph. Most often, it will be possible to continue this incremental process until the puzzle is solved.

Further Explorations

Once you reach the goal and get the lilies to bloom, try to make as many intersecting lines as possible. Is it attainable to have all lines intersecting (no green stems in the pond)?

Mathematical Topics

Planar graphs
Intersections

Transferable Skills

Intuition about nodes and edges

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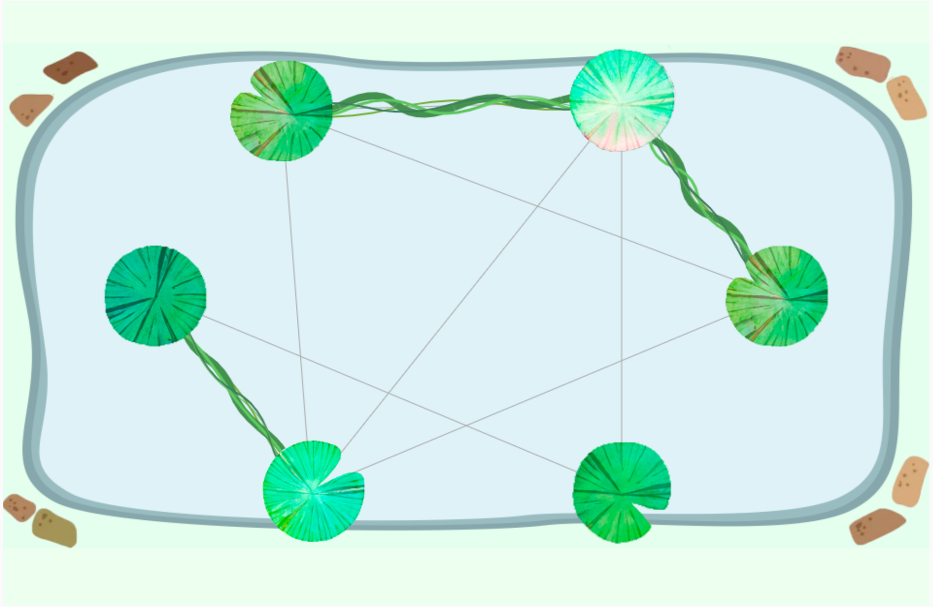
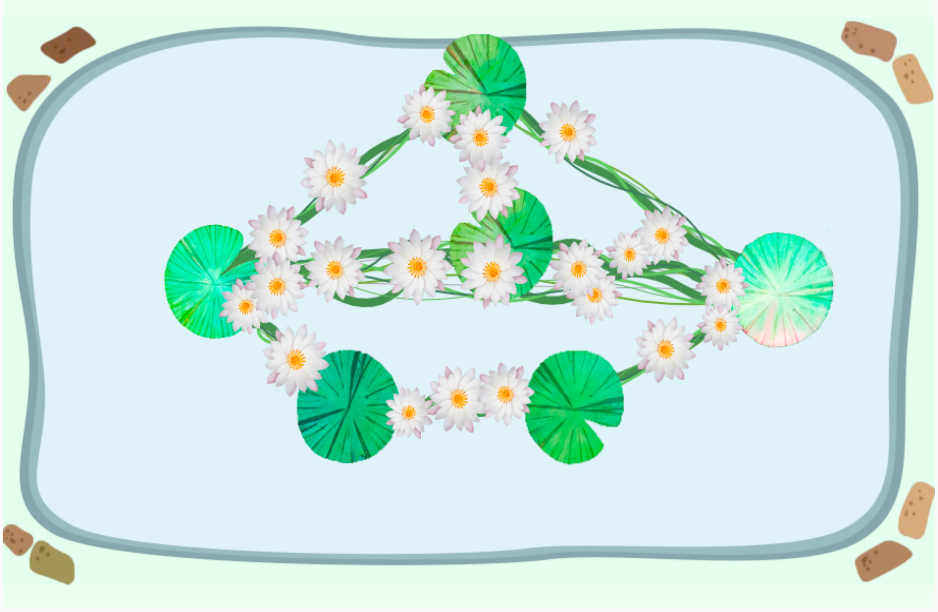
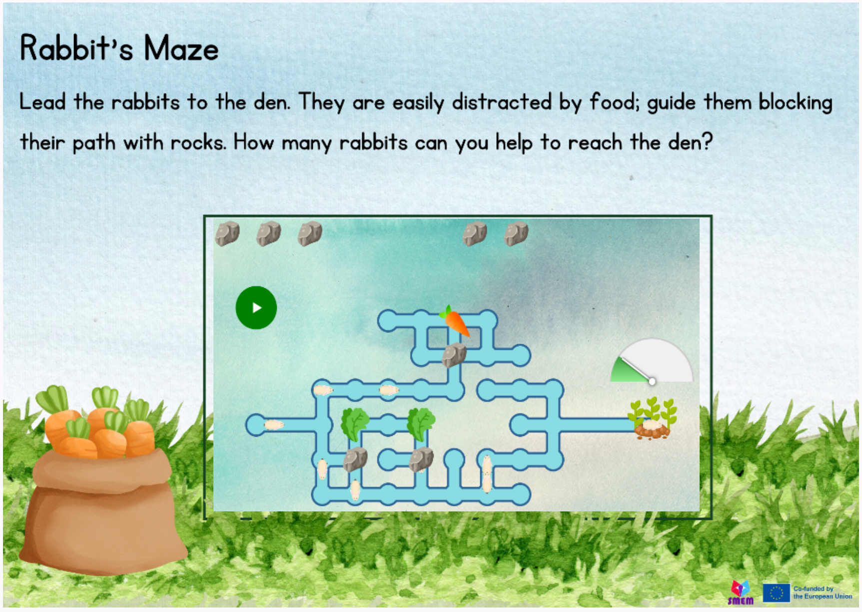




Age Group
3-5

Rabbit's Maze



Material

The board printed on cardboard or laminated paper. Tablet or a computer.

Activity

Several rabbits are in a maze. They need to reach their home (the rabbit hole). If they find food on their way, it will distract them, and escape will last longer. The rabbits move randomly within the labyrinth, but you can block their path by placing a rock on the corners of their path through the maze. Help the rabbits reach their hole by avoiding to let them find the food.

A dial will measure how many rabbits you have helped find their home.

Solution

Block the access to food for the rabbits so they don't get distracted. Block the first food piece they encounter on the way, then continue to block the other distractions.

Once all food is blocked, you can help the rabbits reach the hole faster by blocking their way back to the starting position so they cannot return to the maze start but move towards their hole.

Further Explorations

If all of the rabbits don't manage to escape the maze, play the same game again and try to help more rabbits find their way home.

Try to use as few stones as possible for the activity.

Mathematical Topics

Optimization
Directions
Spatial orientation
Partial success, rate of success

Transferable Skills

Cause and effect logic
Orientation

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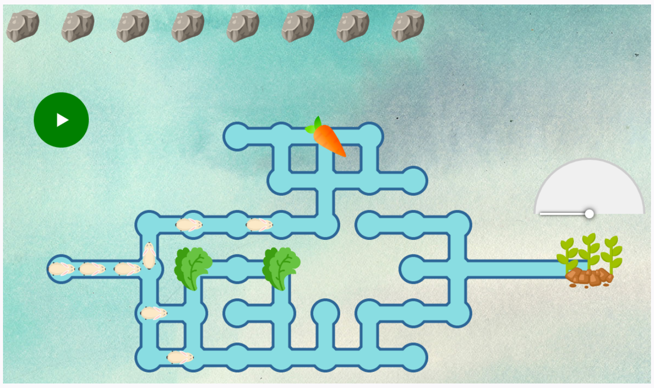
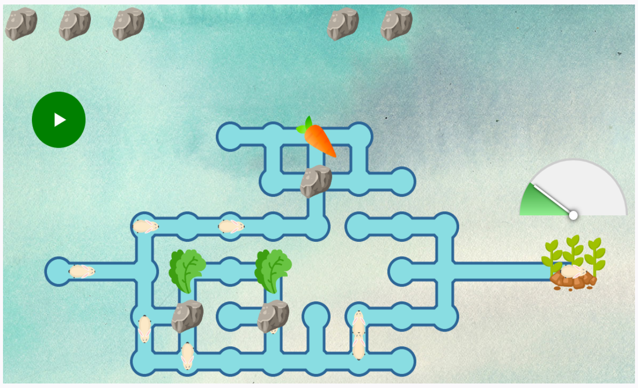
Improvement of a solution





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Singing Birds

Age Group
3-8

Material
The board printed on cardboard or laminated paper. Tablet or a computer.

Activity
The application displays six birds and six mushroom buttons. The rules are the following:
Each bird can be "turned on" (shown in colour, it sings a note) or "off" (shown in grey, it is mute).
Initially, all the birds are silent ("turned off").
Each mushroom button switches the state of some birds, but we do not know in advance which ones.
The goal is to switch all the birds "on" and make them sing a nice musical chord.
Once you solve the puzzle, you can reload the page, and the app will create a new puzzle. The app is programmed to deliver a solvable puzzle.

Solution
There are a few observations that make the puzzle easier to solve:

- The order of pushing mushrooms does not matter, and if you push a mushroom twice, it has no effect.
- Make a table: six rows (mushrooms) and six columns (birds). For each row (mushroom), mark birds which start singing. To find a solution, you have to select some of the mushrooms in a way that in your table, for each column, there is an odd number of marks.

Further Explorations
Once you have found a solution (meaning that all birds are singing) try to find a way back to the original state by pressing the mushroom buttons.
Challenge your friends to find a specific constellation of the birds (for example: "only the second and third birds are singing" or any other state).

Mathematical topics
Binary states

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Transferable Skills

Trial and error
Observation
Memory training
Cause and effect logic

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